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Oscillatory convection in a porous medium heated from below

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The stability of natural convective flow in a porous medium heated both uniformly and non-uniformly from below is studied in order to determine the possibility of oscillatory and other unsteady flows, and to explore the conditions under which they may occur. The results of the numerical work are directly comparable with experiments using a Hele Shaw cell and also, in the uniformly heated case, with the results of Combarnous & Le Fur (1969) and Caltagirone, Cloupeau & Combarnous (1971). It is shown that for the uniformly heated problem there exist, in certain cases, two distinct possible modes of flow, one of which is fluctuating, the other being steady. However in the non-uniformly heated case the boundary conditions force the solution into a unique mode of flow which is regularly oscillatory when there is considerable non-uniformity in the heat input at the lower boundary, provided that the Rayleigh number is sufficiently high.

1. Introduction

The convective flow of fluid through a permeable material has received considerable attention since the early analyses by Horton & Rogers (1945) and Lapwood (1948). The studies of Wooding (1956), Donaldson (1962), Elder (1966*a*, 1967*b*), Chan, Ivey & Barry (1970), Holst & Aziz (1972*a*, *b*) and Palm, Weber & Kvernvold (1972) all led to analyses of this type of flow which indicated that steady-state flow patterns evolved from the initially motionless system and remained unvarying for all subsequent times; Gill (1969) has shown using perturbation analysis that, for an initially uniform horizontal temperature gradient, this must be so.

During preliminary experiments at the University of Auckland, with a Hele Shaw cell to simulate free convection of fluid in a porous medium, it was noticed that under certain conditions the flow became oscillatory. The simple fluid loop considered by Keller (1966), in which periodic oscillations can occur, suggests a possible mechanism for this process. Further, Elder (1966*b*) noticed 'curious results' in numerical solutions for flows in cavities. These were in the form of secondary flows and he speculated that they may have been caused by disturbances inherent in the numerical method exciting other modes of the system. Recently oscillatory convection in a fluid layer has been described very fully by Moore & Weiss (1973). Also, unsteady flows in porous media have been obtained by Combarnous & Le Fur (1969) and Caltagirone *et al.* (1971), who discovered irregular fluctuations in an experimental flow when the lower boundary was presumed to be uniformly heated and the Rayleigh number was larger than a critical value which lies in the range 240–280 depending on the type of porous medium. The present investigation verifies the existence of both regular and irregular fluctuations in convective flow in a porous medium heated from below, determines their nature and considers the processes which may cause them.

The study of incompressible flow through a porous matrix is of particular practical interest in relation to the use of geothermally heated steam as a source of electrical power, for example the Wairakei power scheme in New Zealand. Clearly the possible presence of transiency in such a useful natural resource is significant, and it is for this reason that an attempt is made here to obtain a better understanding of these effects. Since the Wairakei system has been observed for only a short time (on a geophysical time scale), the existence of regular oscillations of low frequency may not have become apparent. Unfortunately, since details of the deep physical system are unknown, it is impossible to perform a quantitative simulation of the flow patterns which exist there. Thus our intention is to generate a flow in a theoretical model and thereby obtain a more general and purely qualitative impression of the phenomenon, although it may be possible to gain some insight as to its magnitude and the observable effects which it may have.

The convection of fluid in a porous medium is also of interest since it is one of the simplest systems exhibiting nonlinear instability.

2. The prototype problem

The Wairakei geothermal area consists of a trough of volcanic debris contained by walls of non-fragmented ignimbrite; thus the model region considered is a long trough of isotropically porous material confined by impermeable and insulating surroundings. The region is considered to be sufficiently long to allow the governing equations to be reduced to two spatial dimensions only, thus resulting in a very worthwhile simplification of their solution. Heat is transported through the region by free convection of the incompressible fluid which completely saturates the medium, or by thermal conduction through the material itself. In the physical system, heat is transferred away from a localized high temperature region caused by magmatic intrusion deep below ground and on reaching the earth's surface produces hydrothermal effects such as steaming ground, fumaroles and hot springs. Elder (1966*a*) suggests that geophysical data indicate that the Wairakei area is a 'wet-convector' in that the medium is completely saturated with liquid and the heating surface is horizontal. It is considered unnecessary to model two-phase flow since, although there exist periodic two-phase disturbances at the surface of geothermal systems, for example geysers and hot lakes, these phenomena are shallow compared with the deep system which is of interest here.

The model region considered is infinitely long, of rectangular cross-section, heated unevenly along its lower surface and insulated on both vertical surfaces (see figure 1). In the calculations carried out here, a square cross-section is used;

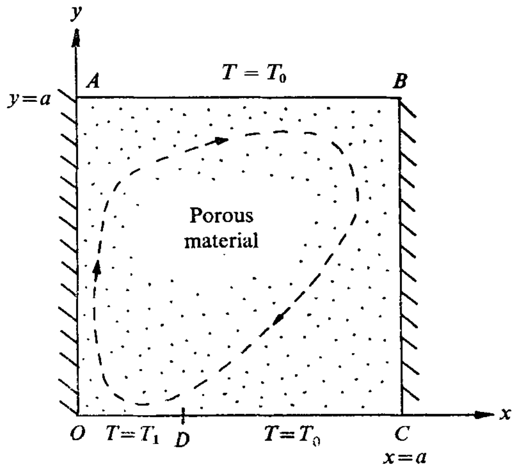

FIGURE 1. Problem boundaries.

however, no loss of generality in the qualitative effects is expected to result from this simplification (see Donaldson 1962).

A portion OD of the lower boundary, representing the heat source, is raised uniformly to a higher temperature T_1, and the remainder DC is maintained at a lower temperature T_0. At the upper boundary heat is lost at a rate proportional to the difference between the ground and air temperatures in the physical system but, since the convective heat loss coefficient is so much larger than that for conductive heat loss, this temperature difference must be small to maintain a balance between the two types of heat transfer, and therefore it is satisfactory to employ a simpler boundary condition, maintaining the surface uniformly at the lower temperature T_0.

3. Governing equations

The motion of the fluid in the region is characterized by the velocity field

$$\mathbf{U}(x,y,t) = (u(x,y,t),\ v(x,y,t), 0),$$

which is governed by the non-dimensional equations

$$\nabla^2\psi = -\theta_X \tag{3.1}$$

and

$$\nabla^2\theta = \theta_\tau + R\,\partial(\psi,\theta), \tag{3.2}$$

where the stream function ψ is defined by

$$\psi_Y = \lambda(a/\kappa R)u, \quad \psi_X = -\lambda(a/\kappa R)v \tag{3.3}$$

and the Jacobian operator $\partial(\psi,\theta) \equiv \psi_Y\theta_X - \psi_X\theta_Y$. Here R is the Rayleigh number

$$R = \frac{g\alpha(T_1-T_0)\,ak}{\nu\kappa}\lambda,$$

where κ is the thermal diffusivity of the fluid-filled medium, k is the permeability of the medium, α is the coefficient of thermal expansion of the fluid, ν is the kinematic viscosity of the fluid and λ is the ratio of the volumetric heat capacity of the fluid to the volumetric heat capacity of the saturated medium. The non-dimensional space, temperature and time variables are defined respectively by

$$X = \frac{x}{a}, \quad Y = \frac{y}{a}, \quad \theta = \frac{T-T_0}{T_1-T_0}, \quad \tau = \frac{\kappa}{a^2}t.$$

The derivation of these equations is well known (see, for example, Wooding 1956; Torrance 1968; Katto & Masuoka 1967). It should be noted that the Boussinesq approximation, that density variations are only significant in their generation of buoyancy forces, has been invoked and also that inertia forces have been ignored. This last assumption can only be made if the Reynolds number for the flow is small.

The Rayleigh number R determines the nature of the flow, which changes from stationary conduction when $R < 4\pi^2$ (Lapwood 1948) to vigorous convection at higher values. It is anticipated that in geothermal regions R lies between 10^3 and 10^4; an estimated value for the Wairakei system is 5000. The present investigation considers values of the Rayleigh number between 0 and 1250, since it will be seen later that it is within this range that the transitions from pure conduction to steady convection and from steady to oscillatory convection occur.

In the model described above, the boundary conditions on θ and ψ are given by

$$\left.\begin{aligned} \theta(X,0,\tau) &= \begin{cases} 1, & 0 < X < f, \\ 0, & f < X < 1, \end{cases} \\ \theta(X,1,\tau) &= 0, \end{aligned}\right\} \tag{3.4a}$$

$$\theta_X(0, Y, \tau) = \theta_X(1, Y, \tau) = 0, \tag{3.4b}$$

and

$$\psi(X, Y, \tau) = 0 \quad \text{on all boundaries.} \tag{3.5}$$

The problem for which $f = 1$ is well known (e.g. Donaldson 1962) and may be solved approximately by using a semi-analytical approach. For example, the spectral method described in Schechter (1967, p. 272) may be used and provides a useful basis for comparison with the more widely applicable methods which are used in this work. This uniformly heated case was considered during this investigation in order to check the accuracy of the numerical algorithms which were used to generate solutions to the non-uniformly heated case ($f < 1$), which were more likely to be susceptible to numerical instabilities because of the higher gradients involved. In particular, a Galerkin (Fourier) technique similar to that described by Orszag (1971) was applied as a check on the finite-difference scheme.

The transient behaviour of similar types of flow has been studied previously by Elder (1967*b*) and more recently by Holst & Aziz (1972*a*, *b*) but in no case were oscillatory solutions found. The investigation by Elder (1967*b*) is of particular interest since his equations are the same as those used here and the boundary conditions are similar. The methods used by the above authors in approximating the heat flow equation are susceptible to nonlinear instability (discussed later) and are not satisfactory for high values of the Rayleigh number, say $R > 200$. For the solution of the Poisson equation Holst & Aziz (1972*a*, *b*) and Elder (1966*b*, 1967*a*, *b*) use the successive over-relaxation method, which although accurate is an iterative and therefore comparatively slow procedure. However the numerical methods used in the present work (a combination of the methods devised by Arakawa 1966 and by Busbee, Golub & Nielsen 1970) were not available at that time and it is largely the remarkable efficiency, accuracy and stability of the methods used here which has enabled the achievement of the long-time solutions which are necessary before the oscillatory solution can be identified. It is interesting to note that the numerical experiments of Moore & Weiss (1973) superseded the results of Fromm (1965) and Veronis (1966) in the description of a hitherto undiscovered oscillatory regime, largely because of the extensive and efficient numerical experiments performed. Moore & Weiss (1973) quote computing times of 5 h on an IBM 360/44 to perform 4000 time steps of their numerical procedure; we were more fortunate in that our problem is much simpler and typical runs of 1500 time steps took 15 min on a Burroughs B 6700.

Steady-state solutions have been obtained by Elder (1966*a*, 1967*a*) to this and very similar problems, but to obtain such a state it was previously assumed that the solution was in fact steady. As Elder (1967*a*) points out, instabilities in the flow cannot be studied by assuming that the temperature field is quasi-steady.

The studies of Caltagirone *et al.* (1971) and Combarnous & Le Fur (1969) indicate possible unsteady flows for the uniformly heated case, but our investigation indicates that an unsteady regime may also be affected by the size f of the heating element.

A fractional value of f destroys the symmetry which is so useful in the consideration of the uniformly heated model, and it is imperative in this case to resort to experimental or numerical methods to obtain a solution. Perturbation analysis is no longer possible, and a simulation of the flow development in time must be performed to determine the stability of the solution. In the sections following, two possible approximate methods of solution are applied: experimental analogy and finite-difference approximation.

4. Experimental solution

The use of a Hele Shaw cell provides a simple analogy with two-dimensional flow through a porous medium (see, for example, Wooding 1960). It is well known (see Yih (1969), p. 382) that the mean flow in a Hele Shaw cell with plate separation b is analogous to the mean two-dimensional seepage flow through a material with permeability $k = \frac{1}{12}b^2$. The two systems are not strictly thermally analogous as heat is inevitably lost to the surroundings through the glass plates of the Hele Shaw cell, and also the heat transfer between the solid and liquid phases is somewhat different to the porous-medium case. However, using an infra-red 'thermovision' camera we found that the mean temperature through the thicknesses of the glass and the water is approximately analogous to the mean temperature at a point in a fluid-saturated porous medium, as reported by Caltagirone *et al.* (1971). The fact that the plates are not completely insulated means that the analogy is not complete; however the experiment is useful for indicating essential features of the flow.

Using a Hele Shaw cell 30×20 cm, a solution similar to the flow in a porous medium at a Rayleigh number of 1600 was generated. The flow of water in the cell was made visible by the injection of dye through small holes in one of the plates, generating streaklines which, owing to the very slow speed of the flow, were unfortunately rather diffuse. The half-heated lower boundary (i.e. $f = 0{\cdot}5$) was achieved by placing a copper heating jacket along the left half of the bottom of the cell and passing water at 40 °C through it at a constant rate for the duration of the experiment. The remaining half of the boundary was similarly cooled. The upper boundary was a free surface, thereby maintaining, first, the low temperature boundary condition by heat loss to the air space, second, the zero normal velocity condition and, third, the constant volume of the system.

After a long period of time, the flow displays a behaviour which is regularly oscillatory, periodically generating 'tongues' of fluid in the descending and ascending regions of the flow. These tongues may be best seen by observing the prominent triangular streakline in the lower left of figure 2 (*a*) (plate 1). In figure 2 (*b*) the triangle is depressed at the top as the descending tongue begins to develop and impressed at the bottom as the ascending tongue forms over the heater. The triangle is further distorted in figure 2 (*c*) as the descending disturbance moves across and down, while the ascending tongue, now quite prominent, moves across the heater towards the left-hand boundary. In figures 2 (*d*) and (*e*) the upper tongue continues its downward flight while the lower one reaches the left-hand boundary and begins to elongate. Finally, in figure 2 (*f*) the ascending disturbance has shot rapidly up the left boundary and the descending one has been completely dissipated. At this time the flow is at the same stage as in figure 2 (*a*).

The period of this oscillation was 900 s in the experiment, which corresponds to a non-dimensional period of 0·003.

We have also modelled the uniformly heated case in the Hele Shaw cell; however experimental work on the uniformly heated model has already been reported by Combarnous & Le Fur (1969) and Caltagirone *et al.* (1971) and its description is not repeated here.

5. Numerical solution

The finite-difference solutions of (3.1) and (3.2) have been of some interest in numerical analysis and fluid mechanics because of the difficulty in representing them satisfactorily in difference form. Torrance (1968) gives a summary of several methods, both explicit and implicit, with special consideration of the parabolic equation describing the transport of temperature (equation (3.2) in this case). The main difficulty in solving this equation in finite-difference form arises in the representation of the term

$$J = \psi_Y \theta_X - \psi_X \theta_Y, \tag{5.1}$$

which is generally known as the advection term (see Crowley 1967). Arakawa (1966) has explained that simple finite-difference approximations using central differences, for example

$$\psi_Y \theta_X = \frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta Y} \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta X}, \tag{5.2}$$

where $$\psi_{i,j} = \psi(i\Delta X, j\Delta Y, \tau), \quad \theta_{i,j} = \theta(i\Delta X, j\Delta Y, \tau)$$

and ΔX and ΔY are spatial increments, cause numerical instability owing to the occurrence of aliasing errors or 'noodling'. Such errors arise because the approximation (5.2) does not conserve the kinetic energy of the system. To avoid aliasing errors Arakawa (1966) developed nine- and thirteen-point representations of J which conserve this quantity and which have a truncation error of the order of the square and fourth power respectively of the spatial difference ΔX (these schemes are accordingly known as the second- and fourth-order Arakawa schemes). Unfortunately both the nine- and thirteen-point templates make implementation difficult as an implicit scheme and so an explicit scheme is usually used. Because of the simple shape of the region and the simple boundary conditions, the numerical solution of the elliptic stream-function equation is much easier to achieve. In this work we use the odd–even reduction method described by Busbee *et al.* (1970), which is a non-iterative scheme for the solution of Poisson's equation on a finite mesh of points. The algorithm is extremely fast but rather elaborate and therefore is not described further here.

Thus the finite-difference form of the equations, using forward differencing in time, becomes

$$[\nabla^2\psi]^n_{i,j} = -[\partial\theta/\partial X]^n_{i,j} \tag{5.3}$$

and

$$(\theta^{n+1}_{i,j} - \theta^n_{i,j})/\Delta\tau = [\nabla^2\theta]^n_{i,j} - RJ^n_{i,j}, \tag{5.4}$$

where the $[\nabla^2\psi]^n_{i,j}$ and $[\nabla^2\theta]^n_{i,j}$ are evaluated using the standard five-point formula and the advection term $J^n_{i,j}$ is evaluated using the fourth-order thirteen-point Arakawa template.

Substitution of the boundary conditions is easily achieved in the solution of (5.4) by direct substitution on the horizontal boundaries and the use of the symmetry of temperatures about the vertical boundaries (on which the normal temperature gradient is zero). The large number of points in the fourth-order Arakawa representation and the use of this image method result in an accurate boundary representation.

We used the fourth-order method in preference to the second-order version because it is the advection term in (5.4) which is dominant in the production of both numerical and physical disturbances. Obviously this term is the most important in this investigation and must be represented as precisely as possible, besides which the fourth-order method is only slightly slower in its computer implementation. Unfortunately it is not possible to apply the thirteen-point template right up to the horizontal boundaries, so for points lying one row in from these boundaries the nine-point template must be used.

6. Fluctuating flows

Uniformly heated boundary

Following the initial checks on the applicability of the numerical scheme to simple cases, by comparison with a Fourier transform method, a set of solutions to the uniformly heated problem was generated to obtain a representation of the fluctuating convective state observed by Combarnous & Le Fur (1969) and

R	Mesh size	Fluctuation time†	Motion observed
50	17 × 17	—	Minimal
250	17 × 17	—	Steady
375	17 × 17	0·0316	Fluctuating
500	17 × 17	0·0200	Fluctuating
750	17 × 17	0·0102	Fluctuating
1000	33 × 33	0·0076	Fluctuating
1250	33 × 33	0·0070	Fluctuating

† The fluctuation time is a characteristic time taken as the interval between two consecutive similar fluctuations (whenever they may occur).

TABLE 1. Summary of results for $f = 1·0$

Caltagirone *et al.* (1971). Beginning the solution at time $\tau = 0$ with the initial conditions that the fluid is everywhere stationary and the lower boundary suddenly raised to a temperature $\theta = 1$, we found that, in contradiction to the results of the authors just mentioned, the flows are multicellular and stable to the small round-off perturbation inherent in the computer at all Rayleigh numbers between 50 and 1250. However, by heating the lower boundary slowly at first, so that a unicellular motion develops, and then more rapidly, a permanently unicellular motion is obtained which is unsteady at Rayleigh numbers larger than approximately 280, a figure which is in agreement with the range of values experimentally observed by Caltagirone *et al.* (1971). This unicellular motion also results if as well as very rapid initial heating an initial unicellular perturbation of the flow is introduced.

At a Rayleigh number of 500, the Nusselt number for the steady tricellular mode is 7·8, whereas for the fluctuating state it varies in time with a mean value of 6·3 and a maximum of 7·0 (considered over a period in which four consecutive relative maxima appear). For steady convection in a porous layer with no restraining side walls the cell width is dependent on the Rayleigh number (see Combarnous 1970; O'Sullivan 1974) with cell widths of 0·5, 0·33 and 0·25 corresponding to Rayleigh numbers of 280, 400 and 700 respectively. The fluctuating solutions exhibit the same generation and dispersion of an additional cell in the otherwise unicellular convective flow and are directly comparable with the previous results of Caltagirone *et al.* (1971). However this behaviour is clearly unlike that observed in the Hele Shaw experiment in that it is not periodic; although two consecutive similar fluctuations may appear the solution is not generally regular.

The results (summarized in table 1) of our simulations of this unstable regime, called by Bories & Combarnous (1973) the fluctuating convective state, suggest that, although the preferred multicellular mode of the system is stable and forms out of the multicellular proto sublayer observed by Elder (1968), it requires only a small initial unicellular perturbation or slow heating to drive it into the unsteady unicellular mode. This is borne out by the experimental solutions of Caltagirone *et al.* (1971), where obviously slight physical disturbances and slow experimental heating produced a similar effect. The fluctuations which later

f	R	Mesh size	τ_P	Motion observed
0·75	500	17 × 17	—	Steady
0·75	750	33 × 33	—	Steady
0·75	1000	33 × 33	—	Steady
0·75	1250	33 × 33	—	Steady
0·5	250	17 × 17	—	Steady
0·5	375	17 × 17	—	Steady
0·5	500	17 × 17	0·0240	Oscillatory
0·5	500	33 × 33	0·0240	Oscillatory
0·5	750	33 × 33	0·0072	Oscillatory
0·5	1000	33 × 33	0·0062	Oscillatory
0·5	1250	33 × 33	0·0051	Oscillatory
0·25	250	17 × 17	—	Steady
0·25	375	17 × 17	—	Steady
0·25	500	17 × 17	0·0194	Oscillatory
0·25	750	33 × 33	0·0089	Oscillatory
0·25	1000	33 × 33	0·0063	Oscillatory
0·25	1250	33 × 33	0·0051	Oscillatory

TABLE 2. Summary of results for $f < 1{\cdot}0$

appear arise from an attempt to resort to the more favoured steady multicellular pattern, which is repressed by the dominant circulation. However, once either the steady or the fluctuating state is well formed, it cannot easily be perturbed and changed completely into the opposite state. Introduction of large random variations in the temperature on the upper and lower boundaries did not effect a change in state.

As will be seen in the following section, altering the length of the heating element affects the 'unfavourability' of the single-celled mode, and solutions to the non-uniformly heated problem clarify the process involved here.

Non-uniformly heated boundary

To simulate the flow in the non-uniformly heated problem, solutions for values of $f = 0{\cdot}25$, $0{\cdot}5$ and $0{\cdot}75$ were generated on a 17×17 mesh for $250 \leqslant R < 750$ and on a 33×33 mesh for $750 \leqslant R \leqslant 1250$, and exhibited the motions summarized in table 2. In some cases a completely regular oscillation with non-dimensional period τ_P appeared. The progress of the solution through a single oscillation of such a flow is illustrated in figure 3, for which the flow is at a Rayleigh number of 750 with $f = 0{\cdot}5$, was generated on a 33×33 mesh and is typical of all the other regular oscillatory solutions observed. The sequence is of four isotherm plots evenly spaced in time and begins at a stage analogous to figure 2(*b*) for the Hele Shaw results. The variation of a typical temperature and of the Nusselt number is illustrated in figure 4 for the same flow for two oscillations.

These oscillatory flows display the same moving 'tongues' as were observed in the experimental solution (figure 2); however, it should be remembered that the flow in the Hele Shaw cell was visualized by the injection of dye through small holes; therefore the representation produced is of streaklines and is not directly comparable with the isotherms illustrated in figure 3. However the superposition

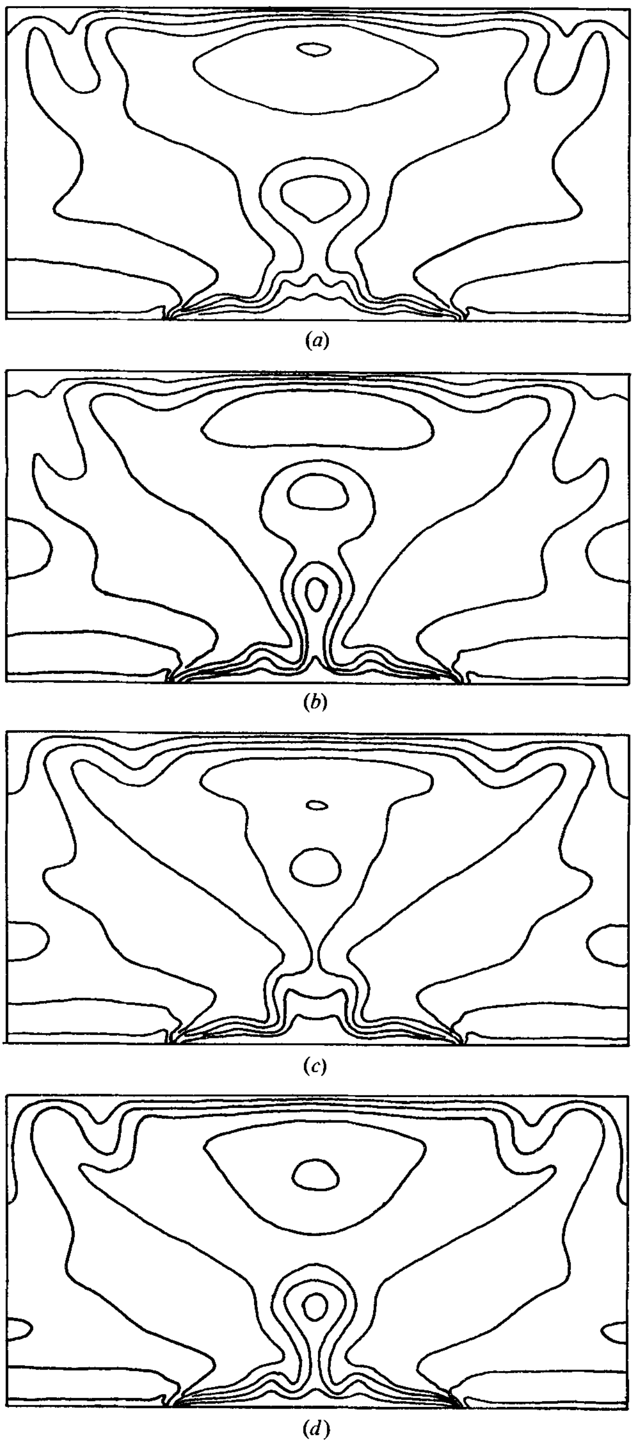

FIGURE 3. Numerical solution for a Rayleigh number of 750 with $f = 0{\cdot}5$, generated on a 33×33 mesh: plot of isotherms during a single oscillation.

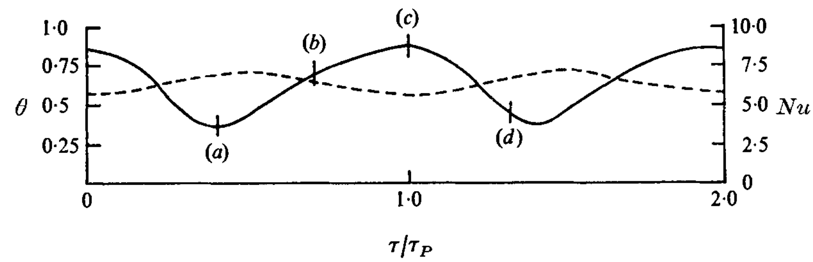

FIGURE 4. Plot of typical temperature (solid line) and Nusselt number (broken line) *vs.* time for two oscillations of the flow in figure 3. Points (*a*)–(*d*) correspond to figures 3(*a*)–(*d*).

of streamlines onto the computed isotherm plot (see figure 5) clearly shows the advection of the fluid 'tongues', the streamlines and isotherms being almost perpendicular in the vicinity (two cells have been placed adjacent to represent flow above a hot 'island'). Particular attention is drawn to the displacement of the streamlines as a disturbance is generated. These displacements are degenerate forms of those observed in the fluctuating state ($f = 1{\cdot}0$), where they act in opposition to the dominant circulation and generate smaller reverse cells before disappearing again.

The variation of the oscillation period τ_P with the Rayleigh number R when $f = 0{\cdot}5$ is plotted in figure 6, alongside the plot of a representative fluctuation time for $f = 1{\cdot}0$; the experimental solution appears as a single point on the same diagram. The value of τ_P obtained in the experiment does not differ greatly from the value expected from a numerical simulation at the same Rayleigh number, the slight difference being of little significance since experimental inaccuracies easily account for the discrepancy (in particular the exact start and finish of a particular oscillation is difficult to establish).

The appearance of identical periodic solutions on 17×17 and on 33×33 meshes at the same Rayleigh number indicates that this behaviour is not merely the result of numerical disturbances; thus the existence of the regular oscillatory solution previously predicted is confirmed. The results demonstrate that large-scale transiency is possible for non-uniform heat distributions at Rayleigh numbers greater than approximately 480 for $f = 0{\cdot}5$ and 450 for $f = 0{\cdot}25$. As was expected, the flow when $f = 0{\cdot}75$ is steady for the values of the Rayleigh number simulated here, and the flow patterns quite different to the other solutions, being tricellular and not single celled (see figure 7). This anomalous set of results is of great significance since, together with the results from the uniformly heated problem, it indicates the cause of the unsteadiness in the other flows. An unheated length of boundary which is similar to the cell width for the steady flow at a particular Rayleigh number forces the favourable steady flow pattern to occur despite any outside perturbation, whereas other heater lengths reinforce the formation of the fluctuating flows of the uniform case and discipline the flow into regular oscillations.

The presence of oscillations such as these in the Wairakei geothermal region would be difficult to establish since the real time constant would be of the order of 1000 years (assuming a depth of 5 km) and there are other complications, for

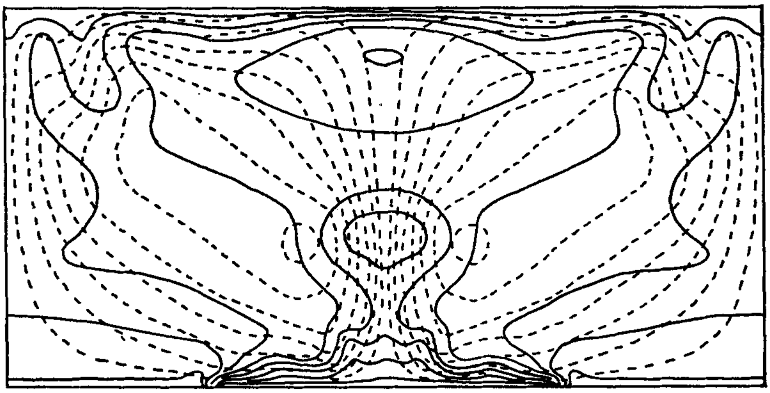

FIGURE 5. Plot of isotherms (solid lines) and streamlines (broken lines) for the flow of figure 3.

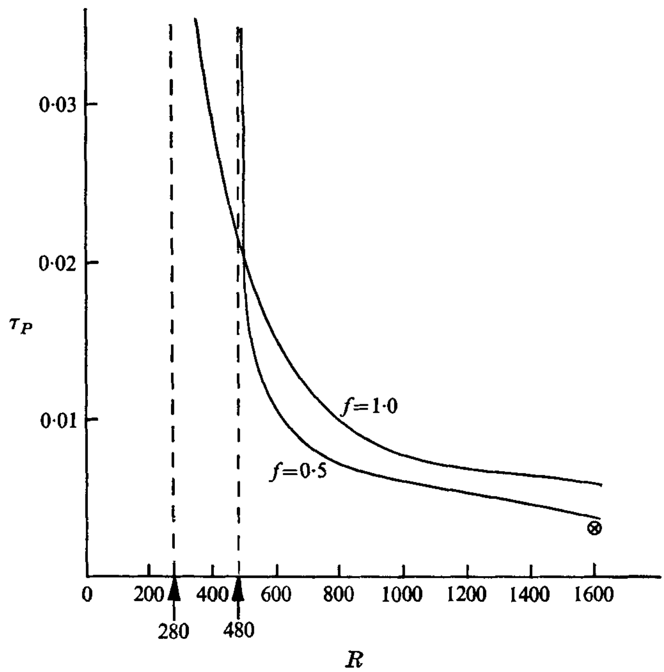

FIGURE 6. Plot of oscillation period τ_P *vs.* Rayleigh number for $f = 0{\cdot}5$, and representative fluctuation time for $f = 1{\cdot}0$. ⊗, experimental solution of figure 2.

instance use of the steam bores and surface water regeneration, which we have ignored so far. The manifestation of oscillatory effects at ground level would be variations in the heat flux from the surface; this is illustrated by the Nusselt-number/time plot in figure 4.

In this work only two-dimensional flows have been considered. It is possible that transitions from two-dimensional rolls to three-dimensional modes may further complicate the problem.

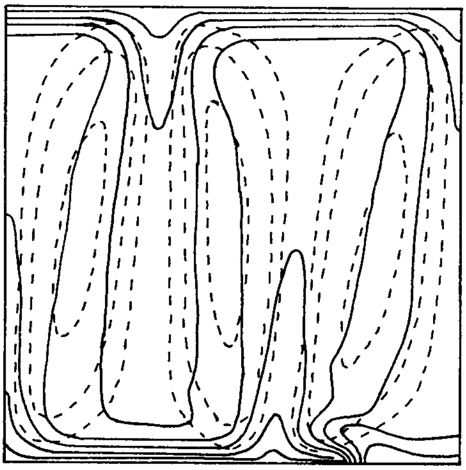

FIGURE 7. Plot of isotherms (solid lines) and streamlines (broken lines) for a flow at a Rayleigh number of 750 with $f = 0{\cdot}75$.

7. Conclusions

When the lower boundary is heated slowly to a final temperature $\theta = 1$, the effect is the same as slowly increasing the Rayleigh number from a low value to the figure under investigation. Therefore, early in the development, at an effectively low Rayleigh number, a convective flow begins in the most favourable mode for that Rayleigh number, namely the unicellular mode. This mode then dominates the later development of the flow until fluctuations start to occur as the system attempts to form a more favourable mode. The same end result is produced by inducing the initial unicellular mode artificially. The appearance of the fluctuations, as has been suggested by Caltagirone *et al.* (1971), is caused by an increase in the local Rayleigh number at the base of the descending column of the unicellular flow above the value at which a convection cell can appear in the small region where the temperature gradient is locally constant. Then an abnormally hot parcel of fluid begins to rise from the surface of the heater, and is moved further by the dominant circulation. A corresponding situation may occur at the upper boundary. As is clearly seen in figure 5 a cold parcel of fluid (indicated by the downward loop in the isotherms) descends more rapidly than its surroundings (indicated by the closeness of the streamlines and their tendency towards the vertical) and preserves its anomalous character as it falls, dwindling slowly owing to thermal diffusion. However, the disturbance is not totally dissipated and on reaching the lower boundary triggers the appearance of the next ascending disturbance. In the uniformly heated problem the position at which the descending disturbance arrives at the lower boundary is variable, thus accounting for the irregular form of the fluctuation; however in the non-uniform case it always arrives at the same place, over the right-hand end of the heater, and so the oscillations are completely regular.

It is seen then that the type of unsteady motion of a fluid flowing through a region of porous material depends on how well the cell shape for steady motion in an unbounded layer fits into the region's boundaries. For Rayleigh numbers greater than 280 (where the steady-flow cell width is 0·5), there is a possibility of either a steady multicellular structure or a fluctuating unicellular structure. Once formed, these two structures are not easily interchangeable, but the system may be assisted into either mode by a suitable perturbation during its early development. If the lower boundary of the region is non-uniformly heated the system is self-restricting and is either stabilized into the steady multicellular flow or regularized into periodic oscillatory flow, depending on how much of the boundary is heated.

Thus the natural convective regime of flow through a porous medium is greatly influenced both by the presence of vertical boundaries and by the type of boundary conditions employed.

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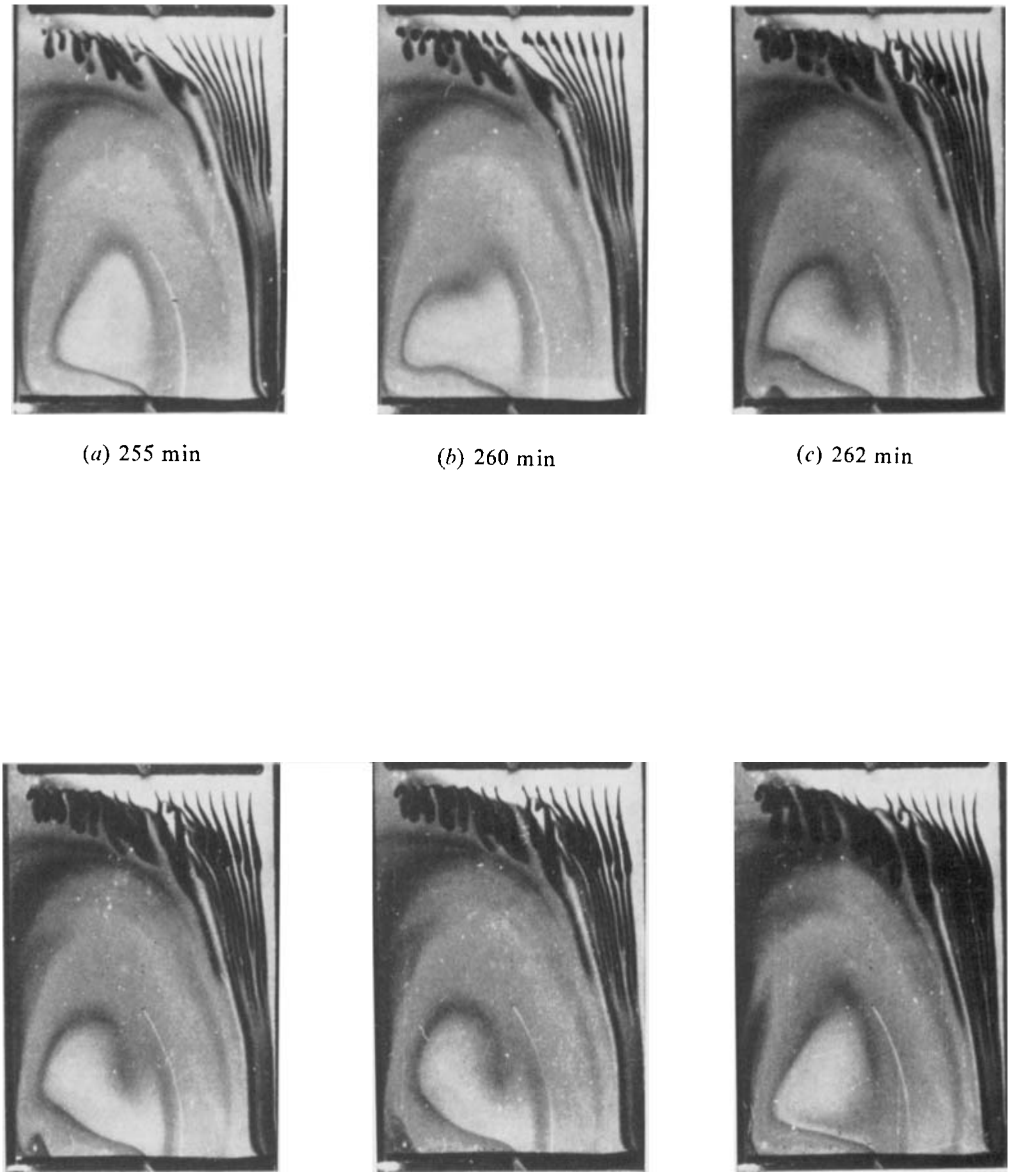

(*a*) 255 min (*b*) 260 min (*c*) 262 min

(*d*) 263 min (*e*) 265 min (*f*) 270 min

FIGURE 2. Experimental solution for a Rayleigh number of 1600 with $f = 0{\cdot}5$: plot of streaklines during a single oscillation.